Reducing the $N = 1$, E_8, 10-dim gauge theory over a modified flag manifold

G. Zoupanos

NTUAthens, MPP-Munich

Bayrischzell, May 2022

1. Short reminder of the Kaluza - Klein programme
2. Higher-Dimensional Unified Gauge Theories and Coset Space Dimensional Reduction (CSDR)
3. The model
4. Embedding in the heterotic $10D$ Superstring

1. Fuzzy extra dimensions → renormalizable realistic 4-d GUTs
2. Reduction of couplings in $\mathcal{N} = 1$ gauge theories → predictive GUTs, Finite Unified Theories, reduced MSSM
3. Noncommutative (fuzzy) Gravity

Kaluza - Klein

- Kaluza-Klein observation: Dimensional Reduction of a pure gravity theory on $M^4 \times S^1$ leads to a $U(1)$ gauge theory coupled to gravity in four dimensions. The extra dimensional gravity provided a geometrical unified picture of gravitation and electromagnetism.
- Generalization to $M^D = M^4 \times B$, with B a compact Riemannian space with a non-abelian isometry group S leads after dim. reduction to gravity coupled to Y-M in 4 dims.

Kerner '68

Cho - Freund '75

Problems

- No classical ground state corresponding to the assumed M^D.
- Adding fermions in the original action, it is impossible to obtain chiral fermions in four dims.

Witten '85

- However by adding suitable matter fields in the original action, in particular Y-M one can have a classical stable ground state of the required form and massless chiral fermions in four dims.

Horvath - Palla - Cremmer - Scherk '77

Original motivation

Use higher dimensions

- to unify the gauge and Higgs sectors
- to unify the fermion interactions with gauge and Higgs fields

⋆ Supersymmetry provides further unification (fermions in adj. reps)

Forgacs - Manton '79, *Manton* '81, *Chapline - Slansky* '82
Kubyshin - Mourao - Rudolph - Volobujev '89
Kapetanakis - Z '92, *Manousselis - Z* '01 −' 08

Further successes

(a) chiral fermions in 4 dims from vector-like reps in the higher dim theory

(b) the metric can be deformed (in certain non-symmetric coset spaces) and more than one scales can be introduced

(c) Wilson flux breaking can be used

(d) Softly broken susy chiral theories in 4 dims can result from a higher dimensional susy theory

Theory in D dims $\rightarrow$ Theory in 4 dims

1. Compactification

$$
\begin{array}{ccccc}
M^D & \rightarrow & M^4 & \times & B \\
| & & | & & | \\
x^M & & x^\mu & & y^a
\end{array}
$$

B - a compact space
$\dim B = D - 4 = d$

2. Dimensional Reduction

Demand that $\mathcal{L}$ is independent of the extra y^a coordinates

- One way: Discard the field dependence on y^a coordinates
- An elegant way: Allow field dependence on y^a and employ a symmetry of the Lagrangian to compensate

  Obvious choice: Gauge Symmetry

Allow a non-trivial dependence on y^a, but impose the condition that a symmetry transformation by an element of the isometry group S of B is compensated by a gauge transformation.

$\Rightarrow$ $\mathcal{L}$ independent of y^a just because is gauge invariant.

Integrate out extra coordinates

CSDR: $B = S/R$

$$S: \quad Q_A = \{Q_i, Q_a\}$$

$$\begin{array}{cc} | & | \\ R & S/R \end{array}$$

$$[Q_i, Q_j] = f_{ij}^k Q_k \, , [Q_i, Q_a] = f_{ia}^b Q_b \, ,$$
$$[Q_a, Q_b] = f_{ab}^i Q_i + f_{ab}^c Q_c \, ,$$

where f_{ab}^c vanishes in symmetric S/R

Consider a Yang-Mills-Dirac theory in D dims based on group G defined on $M^D \to M^4 \times S/R$, $D = 4 + d$

$$g^{MN} = \begin{pmatrix} \eta^{\mu\nu} & 0 \\ 0 & -g^{ab} \end{pmatrix} \qquad \eta^{\mu\nu} = diag(1,-1,-1,-1)$$

$$d = dimS - dimR \qquad g^{ab} - \text{coset space metric}$$

$$A = \int \mathrm{d}^4 x \mathrm{d}^d y \sqrt{-g}\left[-\frac{1}{4} Tr(F_{MN}F_{K\Lambda}) g^{MK} g^{N\Lambda} + \frac{i}{2}\overline{\psi}\Gamma^M D_M \psi \right]$$

$$D_M = \partial_M - \theta_M - A_M \quad , \quad \theta_M = \frac{1}{2}\theta_{MN\Lambda}\Sigma^{N\Lambda}$$

where θ_M is the spin connection of M^D and ψ is in rep F of G

We require that any transformation by an element of S acting on S/R is compensated by gauge transformations.

$$
\begin{aligned}
A_\mu(x,y) =& g(s) A_\mu(x, s^{-1}y) g^{-1}(s) \\
A_a(x,y) =& g(s) J_a{}^b A_b(x, s^{-1}y) g^{-1}(s) \\
& + g(s)\partial_a g^{-1}(s) \\
\psi(x,y) =& f(s) \Omega \psi(x, s^{-1}y) f^{-1}(s)
\end{aligned}
$$

g,f - gauge transformations in the adj, F of G corresponding to the s transformation of S acting on S/R

$J_a{}^b$ - Jacobian for s

Ω - Jacobian + local Lorentz rotation in tangent space

Above conditions imply constraints that D-dims fields should obey.

Solution of constraints:

- 4-dim fields
- Potential
- Remaining gauge invariance

Taking into account all the constraints and integrating out the extra coordinates, we obtain in 4 dims:

$$A = C \int \mathrm{d}^4x \left(-\frac{1}{4}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\sum_a \mathrm{Tr}(D_\mu\phi_a D^\mu\phi^a) + V(\phi) + \frac{i}{2}\bar{\psi}\Gamma^\mu D_\mu\psi - \frac{i}{2}\bar{\psi}\Gamma^a D_a\psi \right)$$

kinetic terms   mass terms

$$D_\mu = \partial_\mu - A_\mu\,,\ D_a = \partial_a - \theta_a - \phi_a\,,\ \theta_a = \frac{1}{2}\theta_{abc}\Sigma^{bc}$$

$C-$ volume of cs , θ_a- spin connection of cs

$$V(\phi) = -\frac{1}{4}g^{ac}g^{bd}\mathrm{Tr}\left\{\left(f_{ab}^C\phi_C - [\phi_a,\phi_b]\right)\left(f_{cd}^D\phi_D - [\phi_c,\phi_d]\right)\right\}$$

$A = 1,\ldots,\mathrm{dim}S,\ f-$ structure constants of S.

Still $V(\phi)$ only formal since ϕ_a must satisfy $f_{ai}^D\phi_D - [\phi_a,\phi_i] = 0$.

1) The 4-dim gauge group

$$H = C_G(R_G)$$

$$\text{i.e.} \quad G \supset R_G \times H$$

where G is the higher-dim group and H is the 4 dim group.

2) Scalar fields

$$S \supset R$$
$$\text{adj}S = \text{adj}R + v$$
$$G \supset R_G \times H$$
$$\text{adj}G \supset (\text{adj}R, 1) + (1, \text{adj}H) + \Sigma(r_i, h_i)$$

If $v = \Sigma s_i$
when $s_i = r_i \Rightarrow \quad h_i$ survives in 4 dims.

3) Fermions

$$G \supset R_G \times H$$
$$F = \sum (t_i, h_i)$$

spinor of $SO(d)$ under R

$$\sigma_d = \sum \sigma_j$$

for every $t_i = \sigma_i \Rightarrow$ h_i survives in 4 dims.

Possible to obtain a chiral theory in 4 dims starting from Weyl fermions in a complex rep.
However, even starting with Weyl (+ Majorana) fermions in vector-like reps of G in $D = 4n + 2$ dims we are also led to a chiral theory.

If D is even:

$$\Gamma^{D+1}\Psi_{\pm} = \pm\Psi_{\pm} \qquad \text{Weyl condition}$$
$$\Psi = \Psi_{+} \oplus \Psi_{-} = \sigma_D + \sigma'_D\,,$$

where σ_D, σ'_D are non-self conjugate spinors of $SO(1, D-1)$.

The $(SU(2) \times SU(2)) \times SO(d)$ branching rule is:

$$\sigma_D = (2, 1; \sigma_d) + (1, 2; \sigma'_d)$$
$$\sigma'_D = (2, 1; \sigma'_d) + (1, 2; \sigma_d)$$

Starting with Dirac fermions

$\rightsquigarrow$ equal number of left and right-handed reps of the 4-dim group H

Weyl condition selects either σ_D or σ'_D

Weyl condition cannot be applied in odd dims. In that case:

$$\sigma_D = (2, 1; \sigma_d) + (1, 2; \sigma_d) \, ,$$

where σ_d is the unique spinor of $SO(d)$

$\rightsquigarrow$ equal number of left and right-handed reps in 4 dims

Most interesting case is when $D = 4n + 2$ and we start with a vectorlike rep. In that case σ_d is non-self-conjugate and $\sigma'_d = \bar{\sigma}_d$.

Then the decomposition of $\sigma_d, \bar{\sigma}_d$ of $SO(d)$ under R is:

$$\sigma_d = \sum \sigma_k \, , \quad \bar{\sigma}_d = \sum \bar{\sigma}_k \, .$$

Then:

$$G \supset R_G \times H$$

vectorlike $\leftarrow F = \sum_i (r_i, h_i) \rightarrow$ either self-conjugate or

have a partner $(\bar{r}_i, \bar{h}_i)$.

Then according to the rule from σ_d we will obtain in 4 dims left-handed fermions $f_L = \sum h_k^L$.

Since σ_d is non-self-conjugate, f_L is non-self-conjugate.

Similarly, from $\bar{\sigma}_d$, we obtain the right-handed rep $\sum \bar{h}_k^R = \sum h_k^L$.

Moreover since F vectorlike, $\bar{h}_k^R \sim h_k^L$, i.e. H is chiral theory with double spectrum.

We can still impose Majorana condition (Weyl and Majorana are compatible in $4n+2$ dims) to eliminate the doubling of the fermion spectrum.

Majorana condition (reverses the sign of all int. qu. nos) forces f_R to be the charge conjugate of f_L.

If F complex $\rightarrow$ chiral theory just $\bar{h}_k^R$ is different from h_k^L.

An easy case in calculating the potential, its minimization and SSB:

If $G \supset S$ $\Rightarrow$ H breaks to $K = C_G(S)$:

$$\begin{array}{ccl} G \supset & S \times K & \leftarrow \text{gauge group after SSB} \\ & \cup \quad \cap & \\ G \supset & R \times H & \leftarrow \text{gauge group in 4 dims} \end{array}$$

But

fermion masses

$$M^2\Psi = D_a D^a \Psi - \frac{1}{4} R\Psi - \frac{1}{2} \underbrace{\Sigma^{ab} F_{ab}}_{=0 \text{ if } S \subset G} \Psi > 0$$

comparable to the compactification scale.

Supersymmetry breaking by dim reduction over symmetric CS (e.g $SO(7)/SO(6)$)

Consider $G = E_8$ in 10 dims with Weyl-Majorana fermions in the adjoint rep of E_8, i.e. a susy E_8.
Embedding of $R = SO(6)$ in E_8 is suggested by the decomposition:

$$E_8 \supset SO(6) \times SO(10)$$
$$248 = (15, 1) + (1, 45) + (6, 10) + (4, 16) + (\overline{4}, \overline{16})$$

$$\mathrm{adj}S = \mathrm{adj}R + v$$
$$21 = 15 + 6 \leftarrow \text{vector}$$

Spinor of $SO(6)$: 4
In 4 dims we obtain a gauge theory based on:

$$H = C_{E_8}(SO(6)) = SO(10)\,,$$

with scalars in 10 and fermions in 16.

- *Theorem*: When S/R symmetric, the potential necessarily leads to spontaneous breakdown of H.

•• Moreover in this case we have:

$$\begin{array}{ccccc} E_8 & \supset & SO(7) & \times & SO(9) \\ & & \cup & & \cap \\ E_8 & \supset & SO(6) & \times & SO(10) \end{array}$$

$\Rightarrow$ Final gauge group after breaking:

$$K = C_{E_8}(SO(7)) = SO(9)$$

CSDR over symmetric coset spaces breaks completely original supersymmetry.

Soft Supersymmetry Breaking by CSDR over non-symmetric CS.

We have examined the dim reduction of a supersymmetric E_8 over the 3 existing 6−dim CS:

$$G_2/SU(3)\,, \quad Sp(4)/(SU(2)\times U(1))_{\text{non-max}}\,, \quad SU(3)/U(1)\times U(1)$$

$\Rightarrow$ Softly Broken Supersymmetric Theories in 4 dims without any further assumption

Non-symmetric CS admit torsion and the two latter more than one radii.

Consider supersymmetric E_8 in 10 dims and $S/R = G_2/SU(3)$.

We use the decomposition:

$$E_8 \supset SU(3) \times E_6$$
$$248 = (8,1) + (1,78) + (3,27) + (\overline{3},\overline{27})$$

and choose $R = SU(3)$

$$\text{adj}S = \text{adj}R + v$$
$$14 = 8 + \underbrace{3 + \bar{3}}_{\text{vector}}$$

Spinor: $1 + 3$ under $R = SU(3)$

$\Rightarrow$ In 4 dim theory: $H = C_{E_8}(SU(3)) = E_6$ with:
scalars in $27 = \beta$ and fermions in $27, 78$

i.e.: spectrum of a supersymmetric E_6 theory in 4 dims.

The Higgs potential of the genuine Higgs β:

$$V(\beta) = 8 - \frac{40}{3}\beta^2 - [4d_{ijk}\beta^i\beta^j\beta^k + h.c.] + \beta^i\beta^j d_{ijk} d^{k\ell m}\beta_\ell\beta_m + \frac{11}{4}\sum_\alpha \beta^i (G^\alpha)_i^j \beta_j \beta^k (G^\alpha)_k^\ell \beta_\ell$$

which obtains F-terms contributions from the superpotential:

$$W(B) = \frac{1}{3} d_{ijk} B^i B^j B^k$$

D-term contributions:

$$\frac{1}{2} D^\alpha D^\alpha\,, \quad D^\alpha = \sqrt{\frac{11}{2}}\beta^i (G^\alpha)_i^j \beta_j$$

The rest terms belong to the SSB part of the Lagrangian:

$$\mathcal{L}^{SSB}_{scalar} = -\frac{1}{R^2}\frac{40}{3}\beta^2 - [4d_{ijk}\beta^i\beta^j\beta^k + h.c.]\frac{g}{R}$$

$$M_{gaugino} = (1 + 3\tau)\frac{6}{\sqrt{3}}\frac{1}{R}$$

Reduction of 10-dim, $\mathcal{N} = 1$, E_8 over $S/R = SU(3)/U(1) \times U(1) \times Z_3$

Irges - Z '11

We use the decomposition:

$$E_8 \supset E_6 \times SU(3) \supset E_6 \times U(1)_A \times U(1)_B$$

and choose $R = U(1)_A \times U(1)_B$,

$$\rightsquigarrow H = C_{E_8}(U(1)_A \times U(1)_B) = E_6 \times U(1)_A \times U(1)_B$$

$$E_8 \supset E_6 \times U(1)_A \times U(1)_B$$

$$\begin{aligned} 248 = & \, 1_{(0,0)} + 1_{(0,0)} + 1_{(3,1/2)} + 1_{(-3,1/2)} \\ & 1_{(0,-1)} + 1_{(0,1)} + 1_{(-3,-1/2)} + 1_{(3,-1/2)} \\ & 78_{(0,0)} + 27_{(3,1/2)} + 27_{(-3,1/2)} + 27_{(0,-1)} \\ & \overline{27}_{(-3,-1/2)} + \overline{27}_{(3,-1/2)} + \overline{27}_{(0,1)} \end{aligned}$$

$$\text{adj}S = \text{adj}R + v \quad \leftarrow \text{vector}$$

$$\Downarrow$$

$$8 = (0,0) + (0,0) + (3, 1/2) + (-3, 1/2) + (0, -1) + (0, 1) + (-3, -1/2) + (3, -1/2)$$

$$SO(6) \supset SU(3) \supset U(1)_A \times U(1)_B$$

$$4 = 1 + 3 = (0,0) + (3, 1/2) + (-3, 1/2) + (0, -1)$$

$$\nwarrow \qquad \nearrow$$

spinor

4-dim theory

$\mathcal{N} = 1, E_6 \times U(1)_A \times U(1)_B$ with chiral supermultiplets:

$A^i : 27_{(3,1/2)}$, $B^i : 27_{(-3,1/2)}$, $C^i : 27_{(0,-1)}$, $A : 1_{(3,1/2)}$, $B : 1_{(-3,1/2)}$, $C : 1_{(0,-1)}$

Scalar potential:

$$\frac{2}{g^2}V = \frac{2}{5}\left(\frac{1}{R_1^4}+\frac{1}{R_2^4}+\frac{1}{R_3^4}\right) + \left(\frac{4R_1^2}{R_2^2R_3^2}-\frac{8}{R_1^2}\right)\alpha^i\alpha_i + \left(\frac{4R_1^2}{R_2^2R_3^2}-\frac{8}{R_1^2}\right)\bar{\alpha}\alpha$$
$$+\left(\frac{4R_2^2}{R_1^2R_3^2}-\frac{8}{R_2^2}\right)\beta^i\beta_i + \left(\frac{4R_2^2}{R_1^2R_3^2}-\frac{8}{R_2^2}\right)\bar{\beta}\beta + \left(\frac{4R_3^2}{R_1^2R_2^2}-\frac{8}{R_3^2}\right)\gamma^i\gamma_i + \left(\frac{4R_3^2}{R_1^2R_2^2}-\frac{8}{R_3^2}\right)\bar{\gamma}\gamma$$
$$+\sqrt{2}80\left[\left(\frac{R_1}{R_2R_3}+\frac{R_2}{R_1R_3}+\frac{R_3}{R_2R_1}\right)d_{ijk}\alpha^i\beta^j\gamma^k + \left(\frac{R_1}{R_2R_3}+\frac{R_2}{R_1R_3}+\frac{R_3}{R_2R_1}\right)\alpha\beta\gamma + h.c\right]$$
$$+\frac{1}{6}\left(\alpha^i(G^\alpha)_i^j\alpha_j + \beta^i(G^\alpha)_i^j\beta_j + \gamma^i(G^\alpha)_i^j\gamma_j\right)^2$$
$$+\frac{10}{6}\left(\alpha^i(3\delta_i^j)\alpha_j + \bar{\alpha}(3)\alpha + \beta^i(-3\delta_i^j)\beta_j + \bar{\beta}(-3)\beta\right)^2$$
$$+\frac{40}{6}\left(\alpha^i(\tfrac{1}{2}\delta_i^j)\alpha_j + \bar{\alpha}(\tfrac{1}{2})\alpha + \beta^i(\tfrac{1}{2}\delta_i^j)\beta_j + \bar{\beta}(\tfrac{1}{2})\beta + \gamma^i(-1\delta_i^j)\gamma^j + \bar{\gamma}(-1)\gamma\right)^2$$
$$+40\alpha^i\beta^j d_{ijk}d^{klm}\alpha_l\beta_m + 40\beta^i\gamma^j d_{ijk}d^{klm}\beta_l\gamma_m + 40\alpha^i\gamma^j d_{ijk}d^{klm}\alpha_l\gamma_m$$
$$+40(\bar{\alpha}\bar{\beta})(\alpha\beta) + 40(\bar{\beta}\bar{\gamma})(\beta\gamma) + 40(\bar{\gamma}\bar{\alpha})(\gamma\alpha)$$

where $\alpha^i, \beta^i, \gamma^i, \alpha, \beta, \gamma$ are the scalar components of A^i, B^i, C^i, A, B, C.

Superpotential: $W(A^i, B^j, C^k, A, B, C) = \sqrt{40} d_{ijk} A^i B^j C^k + \sqrt{40} ABC$

D-terms: $\frac{1}{2} D^\alpha D^\alpha + \frac{1}{2} D_1 D_1 + \frac{1}{2} D_2 D_2$ where:

$$D^\alpha = \frac{1}{\sqrt{3}} \left(\alpha^i (G^\alpha)_i^j \alpha_j + \beta^i (G^\alpha)_i^j \beta_j + \gamma^i (G^\alpha)_i^j \gamma_j \right)$$

$$D_1 = \frac{\sqrt{10}}{3} \left(\alpha^i (3\delta_i^j) \alpha_j + \bar{\alpha}(3)\alpha + \beta^i(-3\delta_i^j)\beta_j + \bar{\beta}(-3)\beta \right)$$

$$D_2 = \frac{\sqrt{40}}{3} \left(\alpha^i (\frac{1}{2}\delta_i^j) \alpha_j + \bar{\alpha}(\frac{1}{2})\alpha + \beta^i(\frac{1}{2}\delta_i^j)\beta_j + \bar{\beta}(\frac{1}{2})\beta + \gamma^i(-1\delta_i^j)\gamma_j + \bar{\gamma}(-1)\gamma \right)$$

Soft scalar supersymmetry breaking terms, $\mathcal{L}^{SSB}_{scalar}$:

$$\left(\frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2} \right) \alpha^i \alpha_i + \left(\frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2} \right) \bar{\alpha}\alpha + \left(\frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2} \right) \beta^i \beta_i +$$
$$\left(\frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2} \right) \bar{\beta}\beta + \left(\frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_2^3} \right) \gamma^i \gamma_i + \left(\frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2} \right) \bar{\gamma}\gamma +$$
$$\sqrt{280} \left[\left(\frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_2 R_1} \right) d_{ijk} \alpha^i \beta^j \gamma^k + \left(\frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_2 R_1} \right) \alpha\beta\gamma + h.c. \right],$$

Gaugino mass, $M = (1 + 3\tau) \frac{R_1^2 + R_2^2 + R_3^2}{8\sqrt{R_1^2 R_2^2 R_3^2}}$, τ torsion coeff.

Potential, $V = V_F + V_D + V_{soft}$

The Wilson flux breaking

$M^4 \times B_o \rightarrow M^4 \times B$, $B = B_o / F^{S/R}$
$F^{S/R}$ - a freely acting discrete symmetry of B_o.

1. B becomes multiply connected
2. For every element $g \in F^{S/R}$,

$$\rightsquigarrow \mathcal{V}_g = Pexp\left(-i \int_{\gamma_g} T^a A^a_M(x) dx^M\right) \in H$$

3. If the contour is non-contractible $\rightsquigarrow$ $\mathcal{V}_g \neq 1$ and then $f(g(x)) = \mathcal{V}_g f(x)$, which leads to a breaking of H to $K' = C_H(T^H)$, where T^H is the image of the homomorphism of $F^{S/R}$ into H.
4. Matter fields invariant under $F^{S/R} \oplus T^H$.

In the case of $SU(3)/U(1) \times U(1)$ a freely acting discrete group is:

$$F^{S/R} = \mathbb{Z}_3 \subset W \, , W = \frac{W_S}{W_R} \, ,$$

$W_{S,R}$: Weyl group of S, R.

$$\rightsquigarrow \gamma_3 = \mathrm{diag}(\mathbb{1}, \omega \, \mathbb{1}, \omega^2 \, \mathbb{1}), \quad \omega = e^{2i\pi/3} \in \mathbb{Z}_3$$

The fields that are invariant under $F^{S/R} \oplus T^H$ survive, i.e.:

$$\begin{array}{l} A_\mu = \gamma_3 A_\mu \gamma_3^{-1} \\ A^i = \gamma_3 A^i \, , \quad B^i = \omega \gamma_3 B^i \, , \quad C^i = \omega^2 \gamma_3 C^i \\ A = A \, , \quad B = \omega B \, , \quad C = \omega^2 C \end{array}$$

$$\rightsquigarrow \mathcal{N} = 1 \, , \quad SU(3)_c \times SU(3)_L \times SU(3)_R \, ,$$

Recall that $\quad 27 = (1, 3, \overline{3}) + (3, \overline{3}, 1) + (\overline{3}, 1, 3)$

with matter superfields in:

$$
\begin{array}{ccc}
(1,3,\overline{3})_{(3,1/2)}, & (3,\overline{3},1)_{(0,-1)}, & (\overline{3},1,3)_{(-3,1/2)} \\
\updownarrow & \updownarrow & \updownarrow
\end{array}
$$

$$
L = \begin{pmatrix} H_d^0 & H_u^+ & \nu_L \\ H_d^- & H_u^0 & e_L \\ \nu_R^c & e_R^c & S \end{pmatrix}, \quad q^c = \begin{pmatrix} d_R^{c1} & u_R^{c1} & D_R^{c1} \\ d_R^{c2} & u_R^{c2} & D_R^{c2} \\ d_R^{c3} & u_R^{c3} & D_R^{c3} \end{pmatrix}, \quad Q = \begin{pmatrix} -d_L^1 & -d_L^2 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 \\ D_L^1 & D_L^2 & D_L^3 \end{pmatrix}
$$

and the surviving singlet

$$\theta \to (1,1,1)_{(3,1/2)} \ .$$

Introducing non-trivial windings in R can appear 3 identical flavours in each of the bifundamental matter superfields and singlet superfield.

Further Gauge Breaking of $SU(3)^3$

Babu - He - Pakvasa '86; Ma - Mondragon - Z '04;
Leontaris - Rizos '06; Sayre - Wiesenfeldt - Willenbrock '06

Two generations of L acquire vevs that break the GUT:

$$\langle L_s^{(3)} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & V \end{pmatrix}, \quad \langle L_s^{(2)} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V & 0 & 0 \end{pmatrix}$$

each one alone is not enough to produce the (MS)SM gauge group:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)$$
$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times SU(2)'_R \times U(1)'$$

Their combination gives the desired breaking:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times U(1)_Y$$

Electroweak breaking then proceeds by:

$$\langle L_s^{(3)} \rangle = \begin{pmatrix} \upsilon_d & 0 & 0 \\ 0 & \upsilon_u & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Choice of Radii

– Soft trilinear terms $\sim \frac{1}{R_i}$

– Soft scalar masses $\sim \frac{1}{R_i^2}$

Manolakos - Patellis - Z '20

Two main possible directions:

- Large $R_i \rightarrow$ calculation of the Kaluza-Klein contributions of the 4D theory
  - × Eigenvalues of the Dirac and Laplace operators unknown.
- Small $R_i \rightarrow$ high scale SUSY breaking
- Small $R_i \sim \frac{1}{M_{GUT}}$ with R_3 slightly different such that

$$m_3^2 \sim -\mathcal{O}(TeV^2), \quad m_{1,2}^2 \sim -\mathcal{O}(M_{GUT}^2), \quad a_{abc} \sim M_{GUT}$$

where $m_{1,2,3}^2$ are the squared soft scalar masses and a_{abc} are the soft trilinear couplings.

  - supermassive squarks
  - TeV-scale sleptons
  - TeV-scale soft Higgs squared masses

Reminder: in this scenario $M_{Comp} = M_{GUT}$

Lepton Yukawas and μ terms

At the GUT scale

$$SU(3)^3 \xrightarrow{V} SU(3)_c \times SU(2)_L \times U(1)_Y$$
$$\xrightarrow{v_{u,d}} SU(3)_c \times U(1)_{em}$$

$$\rightarrow \quad \langle\theta^{(3)}\rangle \sim \mathcal{O}(TeV)\ , \quad \langle\theta^{(1,2)}\rangle \sim \mathcal{O}(M_{GUT})$$

The GUT breaking vevs and the $<\theta^{(1,2)}>$ vevs break the two $U(1)$s.

- The two global $U(1)$s forbid Yukawa terms for leptons
  $\rightarrow$ introduce higher-dimensional operators: $\quad L\overline{e}H_d\left(\frac{\overline{K}}{M}\right)^3$
- μ terms for each generation of Higgs doublets are absent
  $\rightarrow$ solution through higher-dim operators: $\quad H_u^{(3)}H_d^{(3)}\overline{\theta}^{(3)}\frac{\overline{K}}{M}$

– $\overline{K}$ is the vev of the conjugate scalar component of either S, ν_R or θ, or any combination of them

Approximate Scale of Parameters

Parameter	Scale
soft trilinear couplings	$\mathcal{O}(GUT)$
squark masses	$\mathcal{O}(GUT)$
slepton masses	$\mathcal{O}(TeV)$
$\mu^{(3)}$	$\mathcal{O}(TeV)$
unified gaugino mass M_U	$\mathcal{O}(TeV)$

Gauge Unification

There exist three basic scales: M_{GUT}, M_{int} and M_{TeV}.

- Squarks, Higgsinos of the two first families and the new exotic quarks decouple at an intermediate scale M_{int}
- Every other high-scale parameter decouples at M_{GUT}

Concerning the 1-loop gauge couplings:

- $\alpha_{1,2}$ are used as input to determine M_{GUT}
- α_3 is found within 2σ of the experimental value

$$a_s(M_Z) = 0.1218$$

$$a_s^{EXP}(M_Z) = 0.1187 \pm 0.0016$$

Scale	GeV
M_{GUT}	$\sim 10^{15}$
M_{int}	$\sim 10^{14}$
M_{TeV}	~ 1500

✓No proton decay problem due to the global symmetries.

1-loop Results

1-loop β-functions used throughout the analysis that change between the three scales M_{GUT}, M_{int} and M_{TeV}.

$\rightarrow$ $m_b(M_Z)$ and $\hat{m}_t$ are found within 2σ of the experimental values

- $m_b(M_Z) = 3.00$ GeV $\quad$ $m_b^{EXP}(M_Z) = 2.83 \pm 0.10$ GeV
- $\hat{m}_t = 171.6$ GeV $\quad$ $\hat{m}_t^{EXP} = 172.4 \pm 0.7$ GeV

$\rightarrow$ m_h is found within 1σ of the experimental value

- $m_h = 125.18$ GeV $\quad$ $m_h^{EXP} = 125.10 \pm 0.14$ GeV

– Large $\tan\beta \sim 48$

– $M_A > 2000$ GeV ✓

CSDR and the Einstein-Yang-Mills system

EYM theory with cosmological constant in $4+d$ dimensions:

$$L = -\frac{1}{16\pi G}\sqrt{-g}R^{(D)} - \frac{1}{4g^2}\sqrt{-g}F^a_{MN}F^{aMN} - \sqrt{-g}\Lambda$$

The corresponding equations of motion are:

$$D_M F^{MN} = 0\,, \quad R_{MN} - \frac{1}{2}Rg_{MN} = -8\pi G T_{MN}$$

Spontaneous compactification: Solutions of the coupled EYM system corresponding to $M^4 \times B$ - B a coset space and α, β coset indices + demanding M^4 to be flat Minkowski:

$$\Lambda = \frac{1}{4}\mathrm{Tr}(F_{\alpha\beta}F^{\alpha\beta})$$

Λ is absent in 4 dims: eliminates the vacuum energy of the gauge fields

Λ equal to the minimum of the potential of the theory

The potential of the reduced low-energy limit of 10-d heterotic string over $SU(3)/U(1) \times U(1)$

Low-energy effective action of $E_8 \times E_8$ heterotic string (bos part):

$$\mathcal{S}_{het} = \frac{1}{2\kappa^2} \int \mathrm{d}^{10}x \sqrt{-|g|} \left(R - \frac{1}{2}\partial_M \tilde{\Phi} \partial^M \tilde{\Phi} - \frac{e^{-\tilde{\Phi}}}{12} \tilde{H}_{MN\Lambda} \tilde{H}^{MN\Lambda} + \frac{\alpha' e^{-\frac{1}{2}\tilde{\Phi}}}{4} \mathrm{Tr}\, F_{MN} F^{MN} \right)$$

- $\kappa^2 = 8\pi G^{(10)}$ the 10-d gravitational constant
- α' the Regge slope parameter
- R the Ricci scalar of the 10-d (target) space
- $\tilde{\Phi}$ the dilaton scalar field
- $\tilde{H}$ the field strength tensor of the 2-form B_{MN} field
- F the field strength tensor of the $E_8 \times E_8$ gauge field

Also, $g_s^2 = e^{2\tilde{\Phi}_0}$ is the string coupling constant ($\tilde{\Phi}_0$ is the constant mode of the dilaton)

Application of the CSDR over $SU(3)/U(1) \times U(1)$ leads to a $4-d$ scalar potential *Chatzistavrakidis - Z '09*

The contributions of the three sectors after the CSDR:

$$V_{gr} = -\frac{1}{4\kappa^2} e^{-\tilde{\phi}} \left(\frac{6}{R_1^2} + \frac{6}{R_2^2} + \frac{6}{R_3^2} - \frac{R_1^2}{R_2^2 R_3^2} - \frac{R_2^2}{R_1^2 R_3^2} - \frac{R_3^2}{R_1^2 R_2^2} \right)$$

$$V_H = \frac{1}{2\kappa^2} e^{-\tilde{\phi}} \left[\frac{(b_1^2+b_2^2+b_3^2)^2}{(R_1R_2R_3)^2} + \sqrt{2} i \alpha' \frac{1}{R_1R_2R_3} (b_1^2+b_2^2+b_3^2)(d_{ijk}\alpha^i \beta^j \gamma^k - h.c.) \right)$$

$$V_F = \frac{\alpha'}{8\kappa^2} e^{-\frac{\tilde{\phi}}{2}} \left[c + \left(\frac{4R_1^2}{R_2^2R_3^2} - \frac{8}{R_1^2} \right) \alpha^i \alpha_i + \left(\frac{4R_2^2}{R_1^2R_3^2} - \frac{8}{R_2^2} \right) \beta^i \beta_i + \left(\frac{4R_3^2}{R_1^2R_2^2} - \frac{8}{R_3^2} \right) \gamma^i \gamma_i \right.$$

$$+ \sqrt{2} 80 \frac{R_1^2+R_2^2+R_3^2}{R_1R_2R_3} (d_{ijk}\alpha^i\beta^j\gamma^k + h.c.) + \frac{1}{6} \left(\alpha^i (G^\alpha)_i^j \alpha_j + \beta^i (G^\alpha)_i^j \beta_j + \gamma^i (G^\alpha)_i^j \gamma_j \right)^2$$

$$+ 5 \left(\alpha^i \alpha_i - \beta^i \beta_i \right)^2 + \frac{10}{3} \left(\alpha^i \alpha_i + \beta^i \beta_i - 2\gamma^i \gamma_i \right)^2$$

$$\left. + 40 \alpha^i \beta^j d_{ijk} d^{klm} \alpha_l \beta_m + 40 \beta^i \gamma^j d_{ijk} d^{klm} \beta_l \gamma_m + 40 \alpha^i \gamma^j d_{ijk} d^{klm} \alpha_l \gamma_m \right]$$

Possible compensation to the negative gravity contribution by the presence of gauge and 3-form sectors.

Gibbons '84; De Wit - Smit - Dass '87;
Maldacena - Nuñez '01, Manousselis - Prezas - Z '06

THANK YOU!

CORFU2022

Workshop on the Standard Model and Beyond
Workshop on Features of A Quantum de Sitter Universe
Workshop on Holography and the Swampland
Workshop on Tensions in Cosmology
Workshop in Celestial Amplitudes and Flat Space Holography
Workshop on Noncommutative and Generalized Geometry in String Theory, Gauge Theory and Related Physical Models
Workshop on Radiopharmaceutical Therapy
Normal Tissue Effects in the Clinic
RPT-TEC-2022
Workshop on Trends in Hardware and Software for Monitoring and Understanding Earthquake Dynamics

Noncommutativity and gravity

A 4d covariant noncommutative space

Motivation for a 4d covariant nc space

- Constructing field theories on nc spaces is non-trivial: nc deformations break Lorentz invariance
- Special: *covariant* noncommutative spaces
- such an example is the fuzzy sphere (2d space) - coords are identified as rescaled SU(2) generators *Madore '92*

  Hammou-Lagraa-Sheikh Jabbari '02

  Vitale-Wallet '13, Vitale '14

  Jurman-Steinacker '14

  Chatzistavrakidis-Jonke-Jurman-Manolakos-Manousselis-Z '18
- Previous work on 3d nc gravity on the covariant spaces $R^3_\lambda (R^{1,2}_\lambda)$
- Need of 4d covariant nc space to construct a gravity gauge theory

Construction of the 4d covariant nc space

Snyder '47, Yang '47

Kimura '02, Heckman-Verlinde '15

Steinacker '16

Sperling-Steinacker '17,'19
Burić-Madore '14,'15

Manousselis-Manolakos-Z '19,'21

- dS_4 : homogeneous space of constant curvature (positive)
- Described by the embedding $\eta^{AB} X_A X_B = R^2$ into M_5
- Aim for a nc version of dS_4
- Coords must satisfy $[X_a, X_b] = i\theta_{ab}$, with θ_{ab} to be determined
- Analogy to the fuzzy sphere case: identification of the coordinates with generators of the SO(1,4) (isometry group of dS_4)
- BUT: θ_{ab} cannot be assigned to generators of the algebra $\rightarrow$ covariance breaks
- Requiring covariance $\rightarrow$ use a group with larger symmetry $\rightarrow$ minimum extension: SO(1,5)

- The SO(1,5) generators, $J_{MN}, M, N = 0, \ldots, 5$, satisfy the commutation relation:

$$[J_{MN}, J_{P\Sigma}] = i(\eta_{MP}J_{N\Sigma} + \eta_{N\Sigma}J_{MP} - \eta_{NP}J_{M\Sigma} - \eta_{M\Sigma}J_{NP})$$

- Employ the decomposition $SO(1,5) \supset SO(1,4) \supset SO(1,3)$
- Introduce a length parameter λ and define operators as rescalings of the generators
- Thus, the commutation relations regarding all the operators $\Theta_{\mu\nu}, X_\mu, P_\mu, h$ are:

$$[\Theta_{\mu\nu}, \Theta_{\rho\sigma}] = i\hbar(\eta_{\mu\rho}\Theta_{\nu\sigma} + \eta_{\nu\sigma}\Theta_{\mu\rho} - \eta_{\nu\rho}\Theta_{\mu\sigma} - \eta_{\mu\sigma}\Theta_{\nu\rho})\,,$$

$$[P_\mu, P_\nu] = i\frac{\hbar}{\lambda^2}\Theta_{\mu\nu}\,, \qquad [X_\mu, X_\nu] = i\frac{\lambda^2}{\hbar}\Theta_{\mu\nu}\,,$$

$$[P_\mu, h] = -i\frac{\hbar}{\lambda^2}X_\mu\,, \qquad [X_\mu, h] = i\frac{\lambda^2}{\hbar}P_\mu\,,$$

$$[P_\mu, X_\nu] = i\hbar\eta_{\mu\nu}h\,, \qquad [\Theta_{\mu\nu}, h] = 0$$

$$[\Theta_{\mu\nu}, P_\rho] = i\hbar(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu)\,, \quad [\Theta_{\mu\nu}, X_\rho] = i\hbar(\eta_{\mu\rho}X_\nu - \eta_{\nu\rho}X_\mu)$$

- The above relations describe the noncommutative space

Nc gauge theories

- Coord operators satisfy: $[X_\mu, X_\nu] = i\theta_{\mu\nu}$, $\theta_{\mu\nu}$ arbitrary
- Lie-type nc: $[X_\mu, X_\nu] = iC_{\mu\nu}{}^\rho X_\rho$
- Natural intro of nc gauge theories through *covariant nc coordinates*: $\mathcal{X}_\mu = X_\mu + A_\mu$ *Madore-Schraml-Schupp-Wess '00*
- Obeys a covariant gauge transformation rule: $\delta\mathcal{X}_\mu = i[\epsilon, \mathcal{X}_\mu]$
- A_μ transforms in analogy with the gauge connection: $\delta A_\mu = -i[X_\mu, \epsilon] + i[\epsilon, A_\mu]$, ($\epsilon$ - the gauge parameter)
- Definition of a (Lie-type) nc *covariant field strength tensor*: $F_{\mu\nu} = [\mathcal{X}_\mu, \mathcal{X}_\nu] - iC_{\mu\nu\rho}\mathcal{X}_\rho$
- Gauge theory could be abelian or nonabelian:
  - Abelian if ϵ is a function in $\mathcal{A}$
  - Non-Abelian if ϵ is matrix valued $(\mathrm{Mat}(\mathcal{A}))$

Non-Abelian case

▷ *In nonabelian case, where are the gauge fields valued?*

- Let us consider the CR of two elements of an algebra:

$$[\epsilon, A] = [\epsilon^A T^A, A^B T^B] = \frac{1}{2}\{\epsilon^A, A^B\}[T^A, T^B] + \frac{1}{2}[\epsilon^A, A^B]\{T^A, T^B\}$$

- *Not* possible to restrict to a matrix algebra:
  last term neither *vanishes* in nc nor is an *algebra element*
- There are two options to overpass the difficulty:
  - Consider the universal enveloping algebra
  - Extend the generators and/or fix the rep so that the anticommutators close

▷ *We employ the second option*

4-dim noncommutative gravity

Manolakos, Manousselis, Z '19

- Construction of the gravity model on dS_F^4
- Gauging of the isometry group of the space, $SO(1,4)$ viewed as a subgroup of the total $SO(1,5)$ symmetry
- The anticommutators do not close $\rightarrow$ expansion of the algebra + determination of the rep
- Noncommutative gauge theory: $SO(1,5) \times U(1)$ in 4 rep
- The generators are represented by combinations of 4×4 Γ-matrices

- Specifically, the 4×4 generators of the $SO(1,5)$ are:
  - 6 generators of rotations: $M_{ab} = -\frac{i}{4}[\Gamma_a, \Gamma_b]$
  - 4 generators of the conformal boosts: $K_a = \frac{1}{2}\Gamma_a$
  - 4 generators of the translations: $P_a = -\frac{i}{2}\Gamma_a\Gamma_5$
  - 1 generator of the dilatation: $D = -\frac{1}{2}\Gamma_5$
- Also:
  - 1 generator of the $U(1)$: 1
- The definitions of the above generators determine the algebra:

$$[M_{ab}, M_{cd}] = i(\delta_{ac}M_{bd} + \delta_{bd}M_{ac} - \delta_{bc}M_{ad}\delta_{ad}M_{bc})\,, [K_a, P_b] = i\delta_{ab}D$$
$$[K_a, K_b] = iM_{ab}\,, [P_a, P_b] = iM_{ab}\,, [P_a, D] = iK_a\,, [K_a, D] = -iP_a$$
$$[K_a, M_{bc}] = i(\delta_{ac}K_b - \delta_{ab}K_c)\,, [P_a, M_{bc}] = i(\delta_{ac}P_b - \delta_{ab}P_c)\,, [D, M_{ab}] = 0$$

- Gauging procedure → determination of the covariant coordinate: $\hat{X}_\mu = X_\mu \otimes \mathbf{I} + A_\mu(X)$
- The gauge connection, $A_\mu(X)$:

$$A_\mu(X) = e_\mu{}^a(X) \otimes P_a + \omega_\mu{}^{ab}(X) \otimes M_{ab}(X) + b_\mu{}^a(X) \otimes K_a(X) + \tilde{a}_\mu(X) \otimes D + a_\mu(X) \otimes \mathbf{I}$$

- The gauge parameter $\epsilon(x)$:

$$\epsilon = \epsilon_0(X) \otimes \mathbf{I} + \xi^a(X) \otimes K_a + \tilde{\epsilon}_0(X) \otimes D + \lambda^{ab}(X) \otimes M_{ab} + \tilde{\xi}^a(X) \otimes P_a$$

- The field strength tensor, $\mathcal{R}_{\mu\nu}$:

$$\mathcal{R}_{\mu\nu} = [\hat{X}_\mu, \hat{X}_\nu] - \frac{i\lambda^2}{\hbar} \hat{\Theta}_{\mu\nu} \otimes \mathbf{I} \,,$$

- $\hat{\Theta}_{\mu\nu} = \Theta_{\mu\nu} \otimes \mathbf{I} + \mathcal{B}_{\mu\nu}$, and $\mathcal{B}_{\mu\nu}$ is the 2-form gauge field
- $\mathcal{R}_{\mu\nu}$ is valued in the algebra → expands on the generators:

$$\mathcal{R}_{\mu\nu}(X) = R_{\mu\nu}{}^{ab} \otimes M_{ab} + \tilde{R}_{\mu\nu}{}^a \otimes P_a + R_{\mu\nu}{}^a \otimes K_a + \tilde{R}_{\mu\nu} \otimes D + R_{\mu\nu} \otimes \mathbf{I}$$

- Calculation of the transformations of the gauge fields and the component curvature tensors

NC gauge theory and the action

Manolakos, Manousselis, Z '21

- Start with the following action:

$$\mathcal{S} = \text{Tr}\left([X_\mu, X_\nu] - \kappa^2\Theta_{\mu\nu}\right)\left([X_\rho, X_\sigma] - \kappa^2\Theta_{\rho\sigma}\right)\epsilon^{\mu\nu\rho\sigma}$$

- Field equations satisfied by the nc space for $\kappa^2 = i\lambda^2/\hbar$
- Introduce gauge fields as fluctuations:

$$\mathcal{S} = \text{Trtr}\epsilon^{\mu\nu\rho\sigma}\left([X_\mu + A_\mu, X_\nu + A_\nu] - \kappa^2(\Theta_{\mu\nu} + \mathcal{B}_{\mu\nu})\right)\left([X_\rho + A_\rho, X_\sigma + A_\sigma] - \kappa^2(\Theta_{\rho\sigma} + \mathcal{B}_{\rho\sigma})\right)$$

- The above action is written:

$$\mathcal{S} = \text{Trtr}\left([\mathcal{X}_\mu, \mathcal{X}_\nu] - \frac{i\lambda^2}{\hbar}\hat{\Theta}_{\mu\nu}\right)\left([\mathcal{X}_\rho, \mathcal{X}_\sigma] - \frac{i\lambda^2}{\hbar}\hat{\Theta}_{\rho\sigma}\right)\epsilon^{\mu\nu\rho\sigma}$$

$$:= \text{Trtr}\mathcal{R}_{\mu\nu}\mathcal{R}_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} \xrightarrow[\mathcal{X},\hat{\Theta}]{\delta\mathcal{S}} \boxed{\epsilon^{\mu\nu\rho\sigma}\mathcal{R}_{\rho\sigma} = 0\,, \quad \epsilon^{\mu\nu\rho\sigma}[\mathcal{X}_\nu, \mathcal{R}_{\rho\sigma}] = 0}$$

- where we have identified:
  - $\mathcal{X}_\mu = X_\mu + A_\mu$, the covariant coordinate
  - $\hat{\Theta}_{\mu\nu} = \Theta_{\mu\nu} + \mathcal{B}_{\mu\nu}$, the covariant noncommutative tensor
  - $\mathcal{R}_{\mu\nu} = [\mathcal{X}_\mu, \mathcal{X}_\nu] - i\frac{\lambda^2}{\hbar}\hat{\Theta}_{\mu\nu}$, the field strength tensor

Symmetry breaking

Introduction of auxiliary field $\Phi(X)$:

$$\Phi = \tilde{\phi}^a \otimes P_a + \phi^{ab} \otimes M_{ab} + \phi^a \otimes K_a + \phi \otimes \mathbf{I}_4 + \tilde{\phi} \otimes D$$

into the action:

$$\mathcal{S} = \mathrm{Trtr}_G\, \lambda\Phi(X)\mathcal{R}_{\mu\nu}\mathcal{R}_{\rho\sigma}\varepsilon^{\mu\nu\rho\sigma} + \eta\left(\Phi(X)^2 - \lambda^{-2}\mathbf{I}_N \otimes \mathbf{I}_4\right),$$

induces a symmetry breaking:

$$\boxed{\mathrm{S}_{br} = \mathrm{Tr}\left(\frac{\sqrt{2}}{4}\varepsilon_{abcd}R_{\mu\nu}{}^{ab}R_{\rho\sigma}{}^{cd} - 4R_{\mu\nu}\tilde{R}_{\rho\sigma}\right)\varepsilon^{\mu\nu\rho\sigma}}$$

when the auxiliary field is gauge fixed as:

$$\Phi(X) = \tilde{\phi}(X) \otimes D\big|_{\tilde{\phi}=-2\lambda^{-1}} = -2\lambda^{-1}\mathbf{I}_N \otimes D$$

Resulting symmetry: $SO(1,3) \times U(1)$

Comparing to the commutative case

- Ignoring $\mathcal{B}_{\mu\nu}$ and a_μ
- The commutators of functions vanish: $[f(x), g(x)] \rightarrow 0$
- The anticommutators of functions reduce to product: $\{f(x), g(x)\} \rightarrow 2f(x)g(x)$
- The inner derivation becomes: $[X_\mu, f] \rightarrow \partial_\mu f$
- Trace reduces to integration: $\frac{\sqrt{2}}{4}\mathrm{Tr} \rightarrow \int d^4x$
- We also regard the following reparametrizations:

\- $e_\mu{}^a \rightarrow ime_\mu{}^a \,, \quad P_a \rightarrow -\frac{i}{m}P_a \,, \quad \tilde{R}_{\mu\nu}{}^a \rightarrow imT_{\mu\nu}{}^a$

\- $\omega_\mu{}^{ab} \rightarrow -\frac{i}{2}\omega_\mu{}^{ab} \,, \quad M_{ab} \rightarrow 2iM_{ab} \,, \quad R_{\mu\nu}{}^{ab} \rightarrow -\frac{i}{2}R_{\mu\nu}{}^{ab}$

- Similar procedure to the gauge-theoretic approach of Einstein gravity is followed leading to the same results, i.e. Palatini action with cosmological constant

Conclusions & Future plans

- Construction of a noncommutative gauge theory in 4d
- Description of 4d gravity in a regime where coords can be considered nc
- Commutative analogue of the gauge-theoretic approach of 4-d Einstein gravity
- *Next step:* Search for cosmological consequences